



North Carolina Department of Public Instruction

## **INSTRUCTIONAL SUPPORT TOOLS** FOR ACHIEVING NEW STANDARDS

### ***1<sup>st</sup> Grade Mathematics • Unpacked Content***

For the new Common Core State Standards that will be effective in all North Carolina schools in the 2012-13.

This document is designed to help North Carolina educators teach the Common Core State Standards (Standard Course of Study). NCDPI staff are continually updating and improving these tools to better serve teachers.

#### **What is the purpose of this document?**

To increase student achievement by ensuring educators understand specifically what the new standards mean a student must know, understand and be able to do. This document may also be used to facilitate discussion among teachers and curriculum staff and to encourage coherence in the sequence, pacing, and units of study for grade-level curricula. This document, along with on-going professional development, is one of many resources used to understand and teach the CCSS.

#### **What is in the document?**

Descriptions of what each standard means a student will know, understand and be able to do. The “unpacking” of the standards done in this document is an effort to answer a simple question “What does this standard mean that a student must know and be able to do?” and to ensure the description is helpful, specific and comprehensive for educators.

#### **How do I send Feedback?**

We intend the explanations and examples in this document to be helpful and specific. That said, we believe that as this document is used, teachers and educators will find ways in which the unpacking can be improved and made ever more useful. Please send feedback to us at [denise.schulz@dpi.nc.gov](mailto:denise.schulz@dpi.nc.gov) or [kitty.rutherford@dpi.nc.gov](mailto:kitty.rutherford@dpi.nc.gov) and we will use your input to refine our unpacking of the standards. Thank You!

#### **Just want the standards alone?**

You can find the standards alone at <http://corestandards.org/the-standards>

## Standards for Mathematical Practice in First Grade

The Common Core State Standards for Mathematical Practice are practices expected to be integrated into every mathematics lesson for all students Grades K-12. Below are a few examples of how these Practices may be integrated into tasks that students complete.

Practice	Explanation and Example
<b>1. Make Sense and Persevere in Solving Problems.</b>	Mathematically proficient students in First Grade continue to develop the ability to focus attention, test hypotheses, take reasonable risks, remain flexible, try alternatives, exhibit self-regulation, and persevere (Copley, 2010). As the teacher uses thoughtful questioning and provides opportunities for students to share thinking, First Grade students become conscious of what they know and how they solve problems. They make sense of task-type problems, find an entry point or a way to begin the task, and are willing to try other approaches when solving the task. They ask themselves, “Does this make sense?” First Grade students’ conceptual understanding builds from their experiences in Kindergarten as they continue to rely on concrete manipulatives and pictorial representations to solve a problem, eventually becoming fluent and flexible with mental math as a result of these experiences.
<b>2. Reason abstractly and quantitatively.</b>	Mathematically proficient students in First Grade recognize that a number represents a specific quantity. They use numbers and symbols to represent a problem, explain thinking, and justify a response. For example, when solving the problem: “ <i>There are 60 children on the playground. Some children line up. There are 20 children still on the playground. How many children lined up?</i> ” first grade students may write $20 + 40 = 60$ to indicate a Think-Addition strategy. Other students may illustrate a counting-on by tens strategy by writing $20 + 10 + 10 + 10 + 10 = 60$ . The numbers and equations written illustrate the students’ thinking and the strategies used, rather than how to simply compute, and how the story is decontextualized as it is represented abstractly with symbols.
<b>3. Construct viable arguments and critique the reasoning of others.</b>	Mathematically proficient students in First Grade continue to develop their ability to clearly express, explain, organize and consolidate their math thinking using both verbal and written representations. Their understanding of grade appropriate vocabulary helps them to construct viable arguments about mathematics. For example, when justifying why a particular shape isn’t a square, a first grade student may hold up a picture of a rectangle, pointing to the various parts, and reason, “It can’t be a square because, even though it has 4 sides and 4 angles, the sides aren’t all the same size.” In a classroom where risk-taking and varying perspectives are encouraged, mathematically proficient students are willing and eager to share their ideas with others, consider other ideas proposed by classmates, and question ideas that don’t seem to make sense.
<b>4. Model with mathematics.</b>	Mathematically proficient students in First Grade model real-life mathematical situations with a number sentence or an equation, and check to make sure that their equation accurately matches the problem context. They also use tools, such as tables, to help collect information, analyze results, make conclusions, and review their conclusions to see if the results make sense and revising as needed.

<p><b>5. Use appropriate tools strategically.</b></p>	<p>Mathematically proficient students in First Grade have access to a variety of concrete (e.g. 3-dimensional solids, ten frames, number balances, number lines) and technological tools (e.g., virtual manipulatives, calculators, interactive websites) and use them to investigate mathematical concepts. They select tools that help them solve and/or illustrate solutions to a problem. They recognize that multiple tools can be used for the same problem- depending on the strategy used. For example, a child who is in the counting stage may choose connecting cubes to solve a problem. While, a student who understands parts of number, may solve the same problem using ten-frames to decompose numbers rather than using individual connecting cubes. As the teacher provides numerous opportunities for students to use educational materials, first grade students’ conceptual understanding and higher-order thinking skills are developed.</p>
<p><b>6. Attend to precision.</b></p>	<p>Mathematically proficient students in First Grade attend to precision in their communication, calculations, and measurements. They are able to describe their actions and strategies clearly, using grade-level appropriate vocabulary accurately. Their explanations and reasoning regarding their process of finding a solution becomes more precise. In varying types of mathematical tasks, first grade students pay attention to details as they work. For example, as students’ ability to attend to position and direction develops, they begin to notice reversals of numerals and self-correct when appropriate. When measuring an object, students check to make sure that there are not any gaps or overlaps as they carefully place each unit end to end to measure the object (iterating length units). Mathematically proficient first grade students understand the symbols they use (<math>=</math>, <math>&gt;</math>, <math>&lt;</math>) and use clear explanations in discussions with others. For example, for the sentence <math>4 &gt; 3</math>, a proficient student who is able to attend to precision states, “Four is more than 3” rather than “The alligator eats the four. It’s bigger.”</p>
<p><b>7. Look for and make use of structure.</b></p>	<p>Mathematically proficient students in First Grade carefully look for patterns and structures in the number system and other areas of mathematics. For example, while solving addition problems using a number balance, students recognize that regardless whether you put the 7 on a peg first and then the 4, or the 4 on first and then the 7, they both equal 11 (commutative property). When decomposing two-digit numbers, students realize that the number of tens they have constructed ‘happens’ to coincide with the digit in the tens place. When exploring geometric properties, first graders recognize that certain attributes are critical (number of sides, angles), while other properties are not (size, color, orientation).</p>
<p><b>8. Look for and express regularity in repeated reasoning.</b></p>	<p>Mathematically proficient students in First Grade begin to look for regularity in problem structures when solving mathematical tasks. For example, when adding three one-digit numbers and by making tens or using doubles, students engage in future tasks looking for opportunities to employ those same strategies. Thus, when solving <math>8+7+2</math>, a student may say, “I know that 8 and 2 equal 10 and then I add 7 more. That makes 17. It helps to see if I can make a 10 out of 2 numbers when I start.” Further, students use repeated reasoning while solving a task with multiple correct answers. For example, in the task “There are 12 crayons in the box. Some are red and some are blue. How many of each could there be?” First Grade students realize that the 12 crayons could include 6 of each color (<math>6+6 = 12</math>), 7 of one color and 5 of another (<math>7+5 = 12</math>), etc. In essence, students repeatedly find numbers that add up to 12.</p>

## Grade 1 Critical Areas

The Critical Areas are designed to bring focus to the standards at each grade by describing the big ideas that educators can use to build their curriculum and to guide instruction. The Critical Areas for First Grade can be found on page 13 in the *Common Core State Standards for Mathematics*.

- 1. Developing understanding of addition, subtraction, and strategies for addition and subtraction within 20.**

Students develop strategies for adding and subtracting whole numbers based on their prior work with small numbers. They use a variety of models, including discrete objects and length-based models (e.g., cubes connected to form lengths), to model add-to, take-from, put-together, take-apart, and compare situations to develop meaning for the operations of addition and subtraction, and to develop strategies to solve arithmetic problems with these operations. Students understand connections between counting and addition and subtraction (e.g., adding two is the same as counting on two). They use properties of addition to add whole numbers and to create and use increasingly sophisticated strategies based on these properties (e.g., “making tens”) to solve addition and subtraction problems within 20. By comparing a variety of solution strategies, children build their understanding of the relationship between addition and subtraction.
- 2. Developing understanding of whole number relationships and place value, including grouping in tens and ones.**

Students develop, discuss, and use efficient, accurate, and generalizable methods to add within 100 and subtract multiples of 10. They compare whole numbers (at least to 100) to develop understanding of and solve problems involving their relative sizes. They think of whole numbers between 10 and 100 in terms of tens and ones (especially recognizing the numbers 11 to 19 as composed of a ten and some ones). Through activities that build number sense, they understand the order of the counting numbers and their relative magnitudes.
- 3. Developing understanding of linear measurement and measuring lengths as iterating length units.**

Students develop an understanding of the meaning and processes of measurement, including underlying concepts such as iterating (the mental activity of building up the length of an object with equal-sized units) and the transitivity principle for indirect measurement.
- 4. Reasoning about attributes of, and composing and decomposing geometric shapes.**

Students compose and decompose plane or solid figures (e.g., put two triangles together to make a quadrilateral) and build understanding of part-whole relationships as well as the properties of the original and composite shapes. As they combine shapes, they recognize them from different perspectives and orientations, describe their geometric attributes, and determine how they are alike and different, to develop the background for measurement and for initial understandings of properties such as congruence and symmetry.

## Common Core Cluster

### Represent and solve problems involving addition and subtraction.

Students develop strategies for adding and subtracting whole numbers based on their prior work with small numbers. They use a variety of models, including discrete objects and length-based models (e.g., cubes connected to form lengths), to model add-to, take-from, put-together, take-apart, and compare situations to develop meaning for the operations of addition and subtraction, and to develop strategies to solve arithmetic problems with these operations.

An important component of solving problems involving addition and subtraction is the ability to recognize that any given group of objects (up to 10) can be separated into sub groups in multiple ways and remain equivalent in amount to the original group (Ex: A set of 6 cubes can be separated into a set of 2 cubes and a set of 4 cubes and remain 6 total cubes).

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **add, adding to, taking from, putting together, taking apart, comparing, unknown, sum, less than, equal to, minus, subtract, the same amount as, and** (to describe (+) symbol), **counting on, making ten, doubles, equation**

**\*NOTE:** *Subtraction names a missing part. Therefore, the minus sign should be read as “minus” or “subtract” but not as “take away”. Although “take away” has been a typical way to define subtraction, it is a narrow and incorrect definition. (\*Fosnot & Dolk, 2001; Van de Walle & Lovin, 2006)*

Common Core Standard	Unpacking What do these standards mean a child will know and be able to do?									
<p><b>1.OA.1</b> Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.<sup>1</sup></p> <p><sup>1</sup> See Glossary, Table 1</p>	<p>First grade students extend their experiences in Kindergarten by working with numbers to 20 to solve a new type of problem situation: Compare (See <b>Table 1</b> at end of document for examples of all problem types). In a Compare situation, two amounts are compared to find “How many more” or “How many less”.</p> <table border="1" data-bbox="583 885 1902 1414"> <thead> <tr> <th colspan="3" data-bbox="583 885 1902 927">Problem Type: Compare</th> </tr> </thead> <tbody> <tr> <td data-bbox="583 927 999 1130"> <p><u>Difference Unknown:</u> “How many more?” version. Lucy has 7 apples. Julie has 9 apples. How many more apples does Julie have than Lucy?</p> </td> <td data-bbox="999 927 1436 1130"> <p><u>Bigger Unknown:</u> “More” version suggests operation. Julie has 2 more apples than Lucy. Lucy has 7 apples. How many apples does Julie have?</p> </td> <td data-bbox="1436 927 1902 1130"> <p><u>Smaller Unknown:</u> Version with “more”  Mastery expected in Second Grade</p> </td> </tr> <tr> <td data-bbox="583 1130 999 1414"> <p><u>“How many fewer?” version</u> Lucy has 7 apples. Julie has 9 apples. How many fewer apples does Lucy have than Julie? <math>7 + \square = 9</math>  <math>9 - 7 = \square</math></p> </td> <td data-bbox="999 1130 1436 1414"> <p><u>Bigger Unknown:</u> Version with “fewer”  Mastery expected in Second Grade</p> </td> <td data-bbox="1436 1130 1902 1414"> <p><u>Smaller Unknown:</u> “Fewer” version suggests operation. Lucy has 2 fewer apples than Julie. Julie has 9 apples. How many apples does Lucy have?</p> </td> </tr> </tbody> </table> <p>Compare problems are more complex than those introduced in Kindergarten. In order to solve compare problem types, First Graders must think about a quantity that is not physically present and must conceptualize that amount.</p>	Problem Type: Compare			<p><u>Difference Unknown:</u> “How many more?” version. Lucy has 7 apples. Julie has 9 apples. How many more apples does Julie have than Lucy?</p>	<p><u>Bigger Unknown:</u> “More” version suggests operation. Julie has 2 more apples than Lucy. Lucy has 7 apples. How many apples does Julie have?</p>	<p><u>Smaller Unknown:</u> Version with “more”  Mastery expected in Second Grade</p>	<p><u>“How many fewer?” version</u> Lucy has 7 apples. Julie has 9 apples. How many fewer apples does Lucy have than Julie? <math>7 + \square = 9</math>  <math>9 - 7 = \square</math></p>	<p><u>Bigger Unknown:</u> Version with “fewer”  Mastery expected in Second Grade</p>	<p><u>Smaller Unknown:</u> “Fewer” version suggests operation. Lucy has 2 fewer apples than Julie. Julie has 9 apples. How many apples does Lucy have?</p>
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In addition, the language of “how many more” often becomes lost or not heard with the language of ‘who has more’. With rich experiences that encourage students to match problems with objects and drawings can help students master these challenges.

**NOTE:** Although First Grade students should have experiences solving and discussing all 12 problem types located in Table 1, they are not expected to master all types by the end of First Grade due to the high language and conceptual demands of some of the problem types. **Please see Table 1 at the end of this document** for problem types that First Grade Students are expected to master by the end of First Grade. (*Note: this Table is different than the Table 1 in the original glossary found on the CCSS website.*)

First Graders also extend the sophistication of the methods they used in Kindergarten (counting) to add and subtract within this larger range. Now, First Grade students use the methods of counting on, making ten, and doubles +/- 1 or +/- 2 to solve problems.

**Example: Nine bunnies were sitting on the grass. Some more bunnies hopped there. Now, there are 13 bunnies on the grass. How many bunnies hopped over there?**

**Counting On Method**

**Student:** Niiinnnee... holding a finger for each next number counted 10, 11, 12, 13. Holding up her four fingers, 4! 4 bunnies hopped over there.”

**Example: 8 red apples and 6 green apples are on the tree. How many apples are on the tree?**

**Making Tens Method**

**Student:** I broke up 6 into 2 and 4. Then, I took the 2 and added it to the 8. That’s 10. Then I add the 4 to the 10. That’s 14. So there are 14 apples on the tree.

**Example: 13 apples are on the table. 6 of them are red and the rest are green. How many apples are green?**

**Doubles +/- 1 or 2**

**Student:** I know that 6 and 6 is 12. So, 6 and 7 is 13. There are 7 green apples.

In order for students to read and use equations to represent their thinking, they need extensive experiences with addition and subtraction situations in order to connect the experiences with symbols (+, -, =) and equations ( $5 = 3 + 2$ ). In Kindergarten, students demonstrated the understanding of how objects can be joined (addition) and separated (subtraction) by representing addition and subtraction situations using objects, pictures and words. In First Grade, students extend this understanding of addition and subtraction situations to use the addition symbol (+) to represent joining situations, the subtraction symbol (-) to represent separating situations, and the equal sign (=) to represent a relationship regarding quantity between one side of the equation and the other.

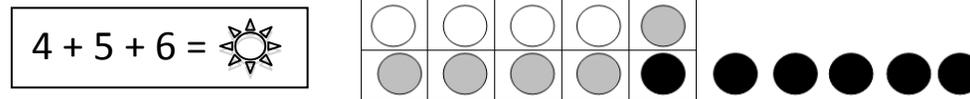
**1.OA.2** Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.

First Grade students solve multi-step word problems by adding (joining) three numbers whose sum is less than or equal to 20, using a variety of mathematical representations.

**Example:** Mrs. Smith has 4 oatmeal raisin cookies, 5 chocolate chip cookies, and 6 gingerbread cookies. How many cookies does Mrs. Smith have?

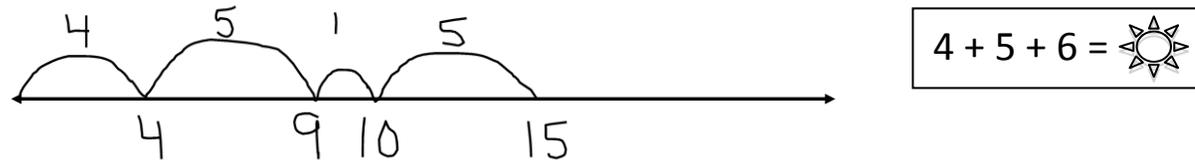
Student A:

I put 4 counters on the Ten Frame for the oatmeal raisin cookies. Then, I put 5 different color counters on the ten frame for the chocolate chip cookies. Then, I put another 6 color counters out for the gingerbread cookies. Only one of the gingerbread cookies fit, so I had 5 leftover. Ten and five more makes 15 cookies. Mrs. Smith has 15 cookies.



Student B:

I used a number line. First I jumped to 4, and then I jumped 5 more. That's 9. I broke up 6 into 1 and 5 so I could jump 1 to make 10. Then, I jumped 5 more and got 15. Mrs. Smith has 15 cookies.



Student C:

I wrote:  $4 + 5 + 6 = \square$ . I know that 4 and 6 equals 10, so the oatmeal raisin and gingerbread equals 10 cookies. Then I added the 5 chocolate chip cookies. 10 and 5 is 15. So, Mrs. Smith has 15 cookies.

## Common Core Cluster

### Understand and apply properties of operations and the relationship between addition and subtraction.

Students understand connections between counting and addition and subtraction (e.g., adding two is the same as counting on two). They use properties of addition to add whole numbers and to create and use increasingly sophisticated strategies based on these properties (e.g., “making tens”) to solve addition and subtraction problems within 20. By comparing a variety of solution strategies, children build their understanding of the relationship between addition and subtraction.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **add, subtract, unknown addend, order, first, second**

#### Common Core Standard

**1.OA.3** Apply properties of operations as strategies to add and subtract.<sup>2</sup>

*Examples: If  $8 + 3 = 11$  is known, then  $3 + 8 = 11$  is also known.*

*(Commutative property of addition.)*

*To add  $2 + 6 + 4$ , the second two numbers can be added to make a ten, so  $2 + 6 + 4 = 2 + 10 = 12$ .*

*(Associative property of addition.)*

<sup>2</sup> Students need not use formal terms for these properties.

#### Unpacking

What do these standards mean a child will know and be able to do?

Elementary students often believe that there are hundreds of isolated addition and subtraction facts to be mastered. However, when students understand the commutative and associative properties, they are able to use relationships between and among numbers to solve problems. First Grade students apply properties of operations as strategies to add and subtract. Students do not use the formal terms “commutative” and “associative”. Rather, they use the understandings of the commutative and associative property to solve problems.

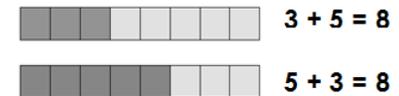
Commutative Property of Addition	Associative Property of Addition
<p>The order of the addends does not change the sum.</p> <p>For example, if <math>8 + 2 = 10</math> is known, then <math>2 + 8 = 10</math> is also known.</p>	<p>The grouping of the 3 or more addends does not affect the sum.</p> <p>For example, when adding <math>2 + 6 + 4</math>, the sum from adding the first two numbers first (<math>2 + 6</math>) and then the third number (4) is the same as if the second and third numbers are added first (<math>6 + 4</math>) and then the first number (2). The student may note that <math>6+4</math> equals 10 and add those two numbers first before adding 2. Regardless of the order, the sum remains 12.</p>

Students use mathematical tools and representations (e.g., cubes, counters, number balance, number line, 100 chart) to model these ideas.

#### Commutative Property Examples:

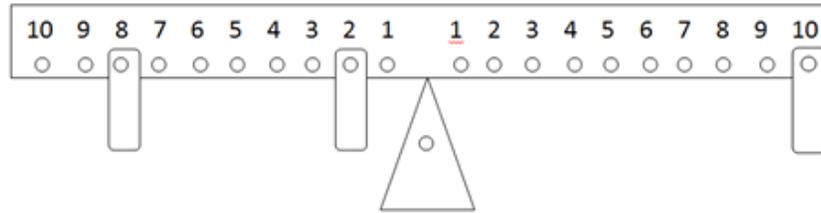
##### Cubes

A student uses 2 colors of cubes to make as many different combinations of 8 as possible. When recording the combinations, the student records that 3 green cubes and 5 blue cubes equals 8 cubes in all. In addition, the student notices that 5 green cubes and 3 blue cubes also equals 8 cubes.



Number Balance

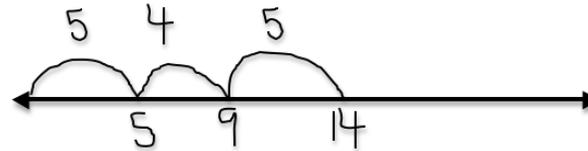
A student uses a number balance to investigate the commutative property. “If 8 and 2 equals 10, then I think that if I put a weight on 2 first this time and then on 8, it’ll also be 10.”



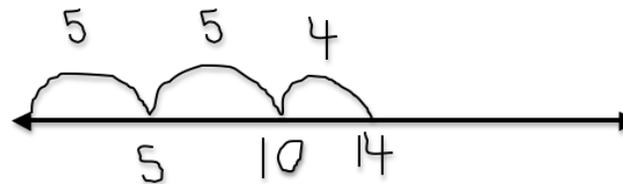
Associative Property Examples:

Number Line:  $\square = 5 + 4 + 5$

**Student A:** First I jumped to 5. Then, I jumped 4 more, so I landed on 9. Then I jumped 5 more and landed on 14.



**Student B:** I got 14, too, but I did it a different way. First I jumped to 5. Then, I jumped 5 again. That’s 10. Then, I jumped 4 more. See, 14!



Mental Math: **There are 9 red jelly beans, 7 green jelly beans, and 3 black jelly beans. How many jelly beans are there in all?**

**Student:** “I know that 7 + 3 is 10. And 10 and 9 is 19. There are 19 jelly beans.”

**1.OA.4** Understand subtraction as an unknown-addend problem.  
*For example, subtract  $10 - 8$  by finding the number that makes 10 when added to 8. Add and subtract within 20.*

First Graders often find subtraction facts more difficult to learn than addition facts. By understanding the relationship between addition and subtraction, First Graders are able to use various strategies described below to solve subtraction problems.

### **For Sums to 10**

#### **\*Think-Addition:**

Think-Addition uses known addition facts to solve for the unknown part or quantity within a problem. When students use this strategy, they think, “What goes with this part to make the total?” The think-addition strategy is particularly helpful for subtraction facts with sums of 10 or less and can be used for sixty-four of the 100 subtraction facts. Therefore, in order for think-addition to be an effective strategy, students must have mastered addition facts first.

For example, when working with the problem  $9 - 5 = \square$ , First Graders think “Five and what makes nine?”, rather than relying on a counting approach in which the student counts 9, counts off 5, and then counts what’s left. When subtraction is presented in a way that encourages students to think using addition, they use known addition facts to solve a problem.

Example:  $10 - 2 = \square$

Student: “2 and what make 10? I know that 8 and 2 make 10. So,  $10 - 2 = 8$ .”

### **For Sums Greater than 10**

The 36 facts that have sums greater than 10 are often considered the most difficult for students to master. Many students will solve these particular facts with Think-Addition (described above), while other students may use other strategies described below, depending on the fact. Regardless of the strategy used, all strategies focus on the relationship between addition and subtraction and often use 10 as a benchmark number.

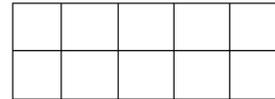
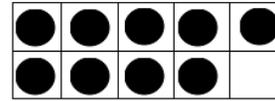
#### **\*Build Up Through 10:**

This strategy is particularly helpful when one of the numbers to be subtracted is 8 or 9. Using 10 as a bridge, either 1 or 2 are added to make 10, and then the remaining amount is added for the final sum.

Example:  $15 - 9 = \square$

Student A: “I’ll start with 9. I need one more to make 10. Then, I need 5 more to make 15. That’s 1 and 5- so it’s 6.  $15 - 9 = 6$ .”

**Student B:** “I put 9 counters on the 10 frame. Just looking at it I can tell that I need 1 more to get to 10. Then I need 5 more to get to 15. So, I need 6 counters.”



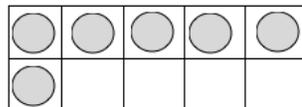
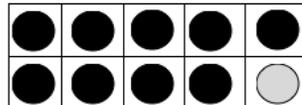
\*Back Down Through 10

This strategy uses take-away and 10 as a bridge. Students take away an amount to make 10, and then take away the rest. It is helpful for facts where the ones digit of the two-digit number is close to the number being subtracted.

Example:  $16 - 7 = \square$

**Student A:** “I’ll start with 16 and take off 6. That makes 10. I’ll take one more off and that makes 9.  $16 - 7 = 9$ .”

**Student B:** “I used 16 counters to fill one ten frame completely and most of the other one. Then, I can take these 6 off from the 2<sup>nd</sup> ten frame. Then, I’ll take one more from the first ten frame. That leaves 9 on the ten frame.”



\*Van de Walle & Lovin, 2006

## Common Core Cluster

### Add and subtract within 20.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **addition, putting together, adding to, making ten, subtraction, taking apart, taking from, equivalent, sum, unknown, equal, equation, counting all, counting on, counting back**

#### Common Core Standard

#### Unpacking

What do these standards mean a child will know and be able to do?

**1.OA.5** Relate counting to addition and subtraction (e.g., by counting on 2 to add 2).

When solving addition and subtraction problems to 20, First Graders often use counting strategies, such as counting all, counting on, and counting back, before fully developing the essential strategy of using 10 as a benchmark number. Once students have developed counting strategies to solve addition and subtraction problems, it is very important to move students toward strategies that focus on composing and decomposing number using ten as a benchmark number, as discussed in 1.OA.6, particularly since counting becomes a hindrance when working with larger numbers. By the end of First Grade, students are expected to use the strategy of 10 to solve problems.

Counting All: Students count all objects to determine the total amount.

Counting On & Counting Back: Students hold a “start number” in their head and count on/back from that number.

Example:  $15 + 2 = \square$

#### Counting All

The student counts out fifteen counters. The student adds two more counters. The student then counts all of the counters starting at 1 (1, 2, 3, 4,...14, 15, 16, 17) to find the total amount.

#### Counting On

Holding 15 in her head, the student holds up one finger and says 16, then holds up another finger and says 17. The student knows that  $15 + 2$  is 17, since she counted on 2 using her fingers.

Example:  $12 - 3 = \square$

#### Counting All

The student counts out twelve counters. The student then removes 3 of them. To determine the final amount, the student counts each one (1, 2, 3, 4, 5, 6, 7, 8, 9) to find out the final amount.

#### Counting Back

Keeping 12 in his head, the student counts backwards, “11” as he holds up one finger; says “10” as he holds up a second finger; says “9” as he holds up a third finger. Seeing that he has counted back 3 since he is holding up 3 fingers, the student states that  $12 - 3 = 9$ .

**1.OA.6** Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g.,  $8 + 6 = 8 + 2 + 4 = 10 + 4 = 14$ ); decomposing a number leading to a ten (e.g.,  $13 - 4 = 13 - 3 - 1 = 10 - 1 = 9$ ); using the relationship between addition and subtraction (e.g., knowing that  $8 + 4 = 12$ , one knows  $12 - 8 = 4$ ); and creating equivalent but easier or known sums (e.g., adding  $6 + 7$  by creating the known equivalent  $6 + 6 + 1 = 12 + 1 = 13$ ).

In First Grade, students learn about and use various strategies to solve addition and subtraction problems. When students repeatedly use strategies that make sense to them, they internalize facts and develop fluency for addition and subtraction within 10. When students are able to demonstrate fluency within 10, they are accurate, efficient, and flexible. First Graders then apply similar strategies for solving problems within 20, building the foundation for fluency to 20 in Second Grade.

Developing Fluency for Addition & Subtraction within 10

**Example:** Two frogs were sitting on a log. 6 more frogs hopped there. How many frogs are sitting on the log now?

Counting- On

I started with 6 frogs and then counted up, Sixxxx.... 7, 8. So there are 8 frogs on the log.  
 $6 + 2 = 8$

Internalized Fact

There are 8 frogs on the log. I know this because 6 plus 2 equals 8.  
 $6 + 2 = 8$

Add and Subtract within 20

**Example:** Sam has 8 red marbles and 7 green marbles. How many marbles does Sam have in all?

Making 10 and Decomposing a Number

I know that 8 plus 2 is 10, so I broke up (decomposed) the 7 up into a 2 and a 5. First I added 8 and 2 to get 10, and then added the 5 to get 15.

$$7 = 2 + 5$$

$$8 + 2 = 10$$

$$10 + 5 = 15$$

Creating an Easier Problem with Known Sums

I broke up (decomposed) 8 into 7 and 1. I know that 7 and 7 is 14. I added 1 more to get 15.

$$8 = 7 + 1$$

$$7 + 7 = 14$$

$$14 + 1 = 15$$

**Example:** There were 14 birds in the tree. 6 flew away. How many birds are in the tree now?

Back Down Through Ten

I know that 14 minus 4 is 10. So, I broke the 6 up into a 4 and a 2. 14 minus 4 is 10. Then I took away 2 more to get 8.

$$6 = 4 + 2$$

$$14 - 4 = 10$$

$$10 - 2 = 8$$

Relationship between Addition & Subtraction

I thought, '6 and what makes 14?'. I know that 6 plus 6 is 12 and two more is 14. That's 8 altogether. So, that means that 14 minus 6 is 8.

$$6 + 8 = 14$$

$$14 - 6 = 8$$

## Common Core Standard and Cluster

### Work with addition and subtraction equations.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **equations, equal, the same amount/quantity as, true, false, addition, putting together, adding to, counting on, making ten, subtract taking apart, taking from, sum, unknown**

Common Core Standard	Unpacking What do these standards mean a child will know and be able to do?
<p><b>1.OA.7</b> Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. For example, which of the following equations are true and which are false? <math>6 = 6</math>, <math>7 = 8 - 1</math>, <math>5 + 2 = 2 + 5</math>, <math>4 + 1 = 5 + 2</math>.</p>	<p>In order to determine whether an equation is true or false, First Grade students must first understand the meaning of the equal sign. This is developed as students in Kindergarten and First Grade solve numerous joining and separating situations with mathematical tools, rather than symbols. Once the concepts of joining, separating, and “the same amount/quantity as” are developed concretely, First Graders are ready to connect these experiences to the corresponding symbols (+, -, =). Thus, students learn that the equal sign does not mean “the answer comes next”, but that the symbol signifies an equivalent relationship that the left side ‘has the same value as’ the right side of the equation.</p> <p>When students understand that an equation needs to “balance”, with equal quantities on both sides of the equal sign, they understand various representations of equations, such as:</p> <ul style="list-style-type: none"><li>• an operation on the left side of the equal sign and the answer on the right side (<math>5 + 8 = 13</math>)</li><li>• an operation on the right side of the equal sign and the answer on the left side (<math>13 = 5 + 8</math>)</li><li>• numbers on both sides of the equal sign (<math>6 = 6</math>)</li><li>• operations on both sides of the equal sign (<math>5 + 2 = 4 + 3</math>).</li></ul> <p>Once students understand the meaning of the equal sign, they are able to determine if an equation is true (<math>9 = 9</math>) or false (<math>9 = 8</math>).</p>
<p><b>1.OA.8</b> Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. <i>For example, determine the unknown number that makes the equation true in each of the equations <math>8 + ? = 11</math>, <math>5 = \_ - 3</math>, <math>6 + 6 = \_</math>.</i></p>	<p>First Graders use their understanding of and strategies related to addition and subtraction as described in 1.OA.4 and 1.OA.6 to solve equations with an unknown. Rather than symbols, the unknown symbols are boxes or pictures.</p> <p><u>Example:</u> <b>Five cookies were on the table. I ate some cookies. Then there were 3 cookies. How many cookies did I eat?</b></p> <p><b>Student A:</b> What goes with 3 to make 5? 3 and 2 is 5. So, 2 cookies were eaten. <b>Student B:</b> Fiiivee, four, three (<i>holding up 1 finger for each count</i>). 2 cookies were eaten (<i>showing 2 fingers</i>). <b>Student C:</b> We ended with 3 cookies. Threeeee, four, five (<i>holding up 1 finger for each count</i>). 2 cookies were eaten (<i>showing 2 fingers</i>).</p> <p><u>Example:</u> <b>Determine the unknown number that makes the equation true. <math>5 - \square = 2</math></b></p> <p><b>Student:</b> 5 minus something is the same amount as 2. Hmmm. 2 and what makes 5? 3! So, 5 minus 3 equals 2.</p>

Now it's true!

## Number and Operations in Base Ten

1.NBT

### Common Core Cluster

#### Extend the counting sequence.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: *number words 0-120*

#### Common Core Standard

**1.NBT.1** Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral.

#### Unpacking

What do these standards mean a child will know and be able to do?

First Grade students rote count forward to 120 by counting on from any number less than 120. First graders develop accurate counting strategies that build on the understanding of how the numbers in the counting sequence are related—each number is one more (or one less) than the number before (or after). In addition, first grade students read and write numerals to represent a given amount.

As first graders learn to understand that the position of each digit in a number impacts the quantity of the number, they become more aware of the order of the digits when they write numbers. For example, a student may write “17” and mean “71”. Through teacher demonstration, opportunities to “find mistakes”, and questioning by the teacher (“I am reading this and it says seventeen. Did you mean seventeen or seventy-one? How can you change the number so that it reads seventy-one?”), students become precise as they write numbers to 120.

## Common Core Cluster

### Understand place value.

Students develop, discuss, and use efficient, accurate, and generalizable methods to add within 100 and subtract multiples of 10. They compare whole numbers (at least to 100) to develop understanding of and solve problems involving their relative sizes. They think of whole numbers between 10 and 100 in terms of tens and ones (especially recognizing the numbers 11 to 19 as composed of a ten and some ones). Through activities that build number sense, they understand the order of the counting numbers and their relative magnitudes.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **tens, ones, bundle, left-overs, singles, groups, greater/less than, equal to, compare**

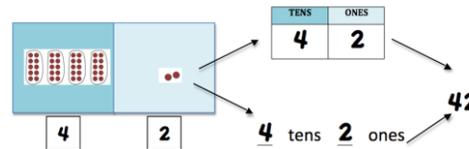
### Common Core Standard

**1.NBT.2** Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases:  
a. 10 can be thought of as a bundle of ten ones — called a “ten.”

### Unpacking

What do these standards mean a child will know and be able to do?

First Grade students are introduced to the idea that a bundle of ten ones is called “a ten”. This is known as unitizing. When First Grade students unitize a group of ten ones as a whole unit (“a ten”), they are able to count groups as though they were individual objects. For example, 4 trains of ten cubes each have a value of 10 and would be counted as 40 rather than as 4. This is a monumental shift in thinking, and can often be challenging for young children to consider a group of something as “one” when all previous experiences have been counting single objects. This is the foundation of the place value system and requires time and rich experiences with concrete manipulatives to develop.



A student’s ability to conserve number is an important aspect of this standard. It is not obvious to young children that 42 cubes is the same amount as 4 tens and 2 left-overs. It is also not obvious that 42 could also be composed of 2 groups of 10 and 22 leftovers. Therefore, first graders require ample time grouping proportional objects (e.g., cubes, beans, beads, ten-frames) to make groups of ten, rather than using pre-grouped materials (e.g., base ten blocks, pre-made bean sticks) that have to be “traded” or are non-proportional (e.g., money).

Example: 42 cubes can be grouped many different ways and still remain a total of 42 cubes.

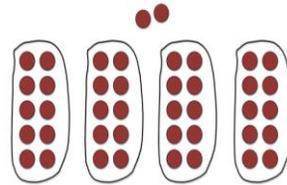


*“We want children to construct the idea that all of these are the same and that the sameness is clearly evident by virtue of the groupings of ten. Groupings by tens is not just a rule that is followed but that any grouping by tens, including all or some of the singles, can help tell how many.” (Van de Walle & Lovin, p. 124)*

As children build this understanding of grouping, they move through several stages: **Counting By Ones**; **Counting by Groups & Singles**; and **Counting by Tens and Ones**.

**Counting By Ones:** At first, even though First Graders will have grouped objects into tens and left-overs, they rely on counting all of the individual cubes by ones to determine the final amount. It is seen as the only way to determine how many.

Example:

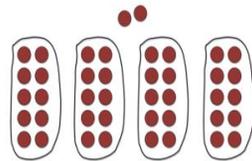


**Teacher:** How many counters do you have?

**Student:** 1, 2, 3, 4, .... 41, 42. I have 42 counters.

**Counting By Groups and Singles:** While students are able to group objects into collections of ten and now tell how many groups of tens and left-overs there are, they still rely on counting by ones to determine the final amount. They are unable to use the groups and left-overs to determine how many.

Example:



**Teacher:** How many counters do you have?

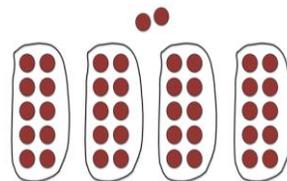
**Student:** I have 4 groups of ten and 2 left-overs.

**Teacher:** Does that help you know how many? How many do you have?

**Student:** Let me see. 1, 2, 3, 4,.... 41, 42. I have 42 counters.

**Counting by Tens & Ones:** Students are able to group objects into ten and ones, tell how many groups and left-overs there are, and now use that information to tell how many. Ex: “I have 3 groups of ten and 4 left-overs. That means that there are 34 cubes in all.” Occasionally, as this stage is becoming fully developed, first graders rely on counting by ones to “really” know that there are 34, even though they may have just counted the total by groups and left-overs.

Example:



**Teacher:** How many counters do you have?

**Student:** I have 4 groups of ten and 2 left-overs.

**Teacher:** Does that help you know how many? How many do you have?

**Student:** Yes. That means that I have 42 counters.

**Teacher:** Are you sure?

**Student:** Um. Let me count just to make sure... 1, 2, 3, 4,... 41, 42. Yes. I was right. There are 42 counters.

**Base Ten Materials: Groupable and Pre-Grouped**

Ample experiences with a variety of groupable materials that are proportional (e.g., cubes, links, beans, beads) and ten frames allow students opportunities to create tens and break apart tens, rather than “trade” one for another. Since students first learning about place value concepts primarily rely on counting, the physical opportunity to build tens helps them to “see” that a “ten stick” has “ten items” within it. Pre-grouped materials (e.g., base ten blocks, bean sticks) are not introduced or used until a student has a firm understanding of composing and decomposing tens. (Van de Walle & Lovin, 2006)

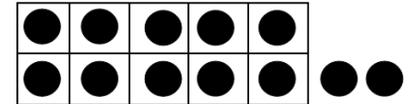
b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.

First Grade students extend their work from Kindergarten when they composed and decomposed numbers from 11 to 19 into ten ones and some further ones. In Kindergarten, everything was thought of as individual units: “ones”. In First Grade, students are asked to unitize those ten individual ones as a whole unit: “one ten”. Students in first grade explore the idea that the teen numbers (11 to 19) can be expressed as *one* ten and some leftover ones. Ample experiences with a variety of groupable materials that are proportional (e.g., cubes, links, beans, beads) and ten frames help students develop this concept.

**Example:** Here is a pile of 12 cubes. Do you have enough to make a ten? Would you have any leftover? If so, how many leftovers would you have?

**Student A**

I filled a ten frame to make one ten and had two counters left over.  
I had enough to make a ten with some leftover.  
The number 12 has 1 ten and 2 ones.



**Student B**

I counted out 12 cubes. I had enough to make 10. I now have 1 ten and 2 cubes left over. So the number 12 has 1 ten and 2 ones.



In addition, when learning about forming groups of 10, First Grade students learn that a numeral can stand for many different amounts, depending on its position or place in a number. This is an important realization as young children begin to work through reversals of digits, particularly in the teen numbers.

**Example: Comparing 19 to 91**

**19**

**91**

**Teacher:** Are these numbers the same or different?

**Students:** Different!

**Teacher:** Why do you think so?

**Students:** Even though they both have a one and a nine, the top one is nineteen. The bottom one is ninety-one.

**Teacher:** Is that true some of the time, or all of the time? How do you know? Teacher continues discussion.

<p>c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).</p>	<p>First Grade students apply their understanding of groups of ten as stated in 1.NBT.2b to decade numbers (e.g. 10, 20, 30, 40). As they work with groupable objects, first grade students understand that 10, 20, 30...80, 90 are comprised of a certain amount of groups of tens with none left-over.</p>
<p><b>1.NBT.3</b> Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols <math>&gt;</math>, <math>=</math>, and <math>&lt;</math>.</p>	<p>First Grade students use their understanding of groups and order of digits to compare two numbers by examining the amount of tens and ones in each number. After numerous experiences verbally comparing two sets of objects using comparison vocabulary (e.g., 42 is more than 31. 23 is less than 52, 61 is the same amount as 61.), first grade students connect the vocabulary to the symbols: greater than (<math>&gt;</math>), less than (<math>&lt;</math>), equal to (<math>=</math>).</p> <p><u>Example: Compare these two numbers. 42 __ 45</u></p> <div style="display: flex; justify-content: space-around; margin-top: 10px;"> <div data-bbox="611 532 1241 716" style="border: 1px solid black; padding: 5px; width: 45%;"> <p><b>Student A</b>  42 has 4 tens and 2 ones. 45 has 4 tens and 5 ones. They have the same number of tens, but 45 has more ones than 42. So, 42 is less than 45.  <math>42 &lt; 45</math></p> </div> <div data-bbox="1287 532 1883 716" style="border: 1px solid black; padding: 5px; width: 45%;"> <p><b>Student B</b>  42 is less than 45. I know this because when I count up I say 42 before I say 45.  <math>42 &lt; 45</math>  This says 42 is less than 45.</p> </div> </div>

## Common Core Cluster

### Use place value understanding and properties of operations to add and subtract.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **ones, tens, add, subtract, reason, more, less**

#### Common Core Standard

**1.NBT.4** Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.

#### Unpacking

What do these standards mean a child will know and be able to do?

First Grade students use concrete materials, models, drawings and place value strategies to add within 100. They do so by being flexible with numbers as they use the base-ten system to solve problems. The standard algorithm of carrying or borrowing is neither an expectation nor a focus in First Grade. Students use strategies for addition and subtraction in Grades K-3. By the end of Third Grade students use a range of algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction to fluently add and subtract within 1000. Students are expected to fluently add and subtract multi-digit whole numbers using the standard algorithm by the end of Grade 4.

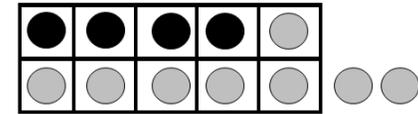
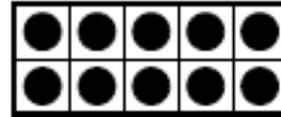
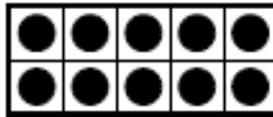
Example: **24 red apples and 8 green apples are on the table. How many apples are on the table?**

#### Student A:

I used ten frames. I put 24 chips on 3 ten frames. Then, I counted out 8 more chips. 6 of them filled up the third ten frame. That meant I had 2 left over. 3 tens and 2 left over. That's 32. So, there are 32 apples on the table.

$$24 + 6 = 30$$

$$30 + 2 = 32$$

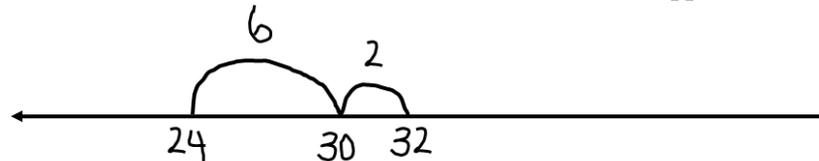


#### Student B:

I used an open number line. I started at 24. I knew that I needed 6 more jumps to get to 30. So, I broke apart 8 into 6 and 2. I took 6 jumps to land on 30 and then 2 more. I landed on 32. So, there are 32 apples on the table.

$$24 + 6 = 30$$

$$30 + 2 = 32$$



#### Student C:

I turned 8 into 10 by adding 2 because it's easier to add. So, 24 and ten more is 34. But, since I added 2 extra, I had to take them off again. 34 minus 2 is 32. There are 32 apples on the table.

$$8 + 2 = 10$$

$$24 + 10 = 34$$

$$34 - 2 = 32$$

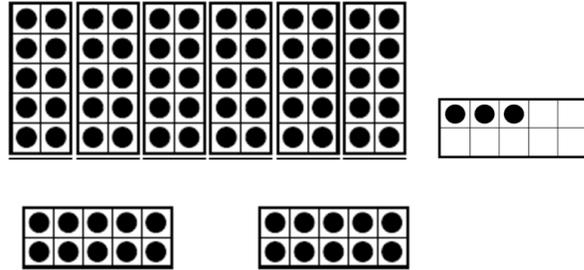
**Example:** 63 apples are in the basket. Mary put 20 more apples in the basket. How many apples are in the basket?

**Student A:**

I used ten frames. I picked out 6 filled ten frames. That's 60. I got the ten frame with 3 on it. That's 63. Then, I picked one more filled ten frame for part of the 20 that Mary put in. That made 73. Then, I got one more filled ten frame to make the rest of the 20 apples from Mary. That's 83. So, there are 83 apples in the basket.

$$63 + 10 = 73$$

$$73 + 10 = 83$$



**Student B:**

I used a hundreds chart. I started at 63 and jumped down one row to 73. That means I moved 10 spaces. Then, I jumped down one more row (that's another 10 spaces) and landed on 83. So, there are 83 apples in the basket.

$$63 + 10 = 73$$

$$73 + 10 = 83$$

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

**Student C:**

I knew that 10 more than 63 is 73. And 10 more than 73 is 83. So, there are 83 apples in the basket.

$$63 + 10 = 73$$

$$73 + 10 = 83$$

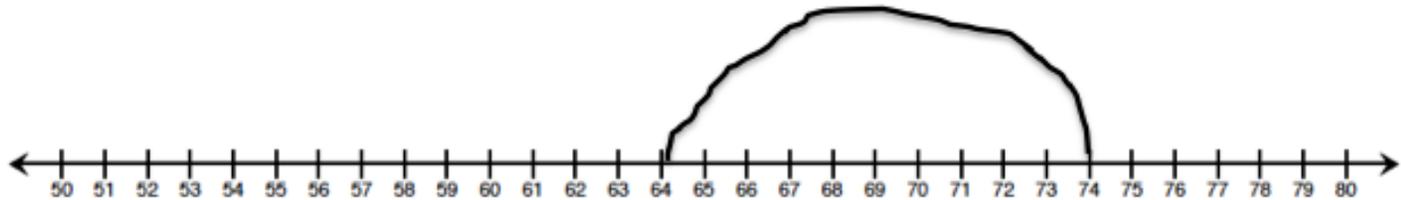
**1.NBT.5** Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.

First Graders build on their counting by tens work in Kindergarten by mentally adding ten more and ten less than any number less than 100. First graders are not expected to compute differences of two-digit numbers other than multiples of ten. Ample experiences with ten frames and the number line provide students with opportunities to think about groups of ten, moving them beyond simply rote counting by tens on and off the decade. Such representations lead to solving such problems mentally.

Example: **There are 74 birds in the park. 10 birds fly away. How many birds are in the park now?**

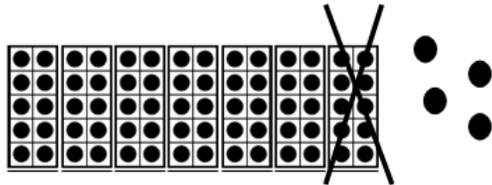
**Student A**

I thought about a number line. I started at 74. Then, because 10 birds flew away, I took a leap of 10. I landed on 64. So, there are 64 birds left in the park.



**Student B**

I pictured 7 ten frames and 4 left over in my head. Since 10 birds flew away, I took one of the ten frames away. That left 6 ten frames and 4 left over. So, there are 64 birds left in the park.



**Student C**

I know that 10 less than 74 is 64. So there are 64 birds in the park.

**1.NBT.6** Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

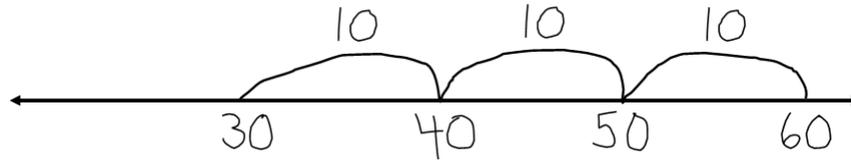
First Grade students use concrete models, drawings and place value strategies to subtract multiples of 10 from decade numbers (e.g., 30, 40, 50). They often use similar strategies as discussed in 1.OA.4.

Example: **There are 60 students in the gym. 30 students leave. How many students are still in the gym?**

**Student A**

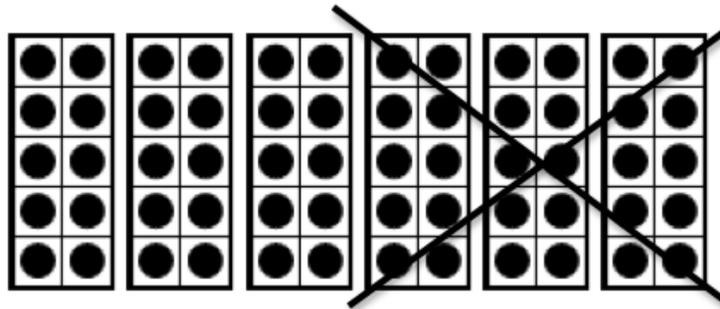
I used a number line. I started at 60 and moved back 3 jumps of 10 and landed on 30. There are 30 students left.

$$\begin{aligned} 60 - 10 &= 50 \\ 50 - 10 &= 40 \\ 40 - 10 &= 30 \end{aligned}$$



**Student B**

I used ten frames. I had 6 ten frames- that's 60. I removed three ten frames because 30 students left the gym. There are 30 students left in the gym.



$$60 - 30 = 30$$

**Student C**

I thought, "30 and what makes 60?". I know 3 and 3 is 6. So, I thought that 30 and 30 makes 60. There are 30 students still in the gym.

$$30 + 30 = 60$$

**Common Core Cluster**

**Measure lengths indirectly and by iterating length units.**

Students develop an understanding of the meaning and processes of measurement, including underlying concepts such as iterating (the mental activity of building up the length of an object with equal-sized units) and the transitivity principle for indirect measurement.<sup>1</sup>

<sup>1</sup>Students should apply the principle of transitivity of measurement to make indirect comparisons, but they need not use this technical term.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **measure, order, length, height, more, less, longer than, shorter than, first, second, third, gap, overlap, about, a little less than, a little more than**

**Common Core Standard**

**Unpacking**

What do these standards mean a child will know and be able to do?

**1.MD.1** Order three objects by length; compare the lengths of two objects indirectly by using a third object.

First Grade students continue to use direct comparison to compare lengths. *Direct* comparison means that students compare the amount of an attribute in two objects without measurement.

Example: **Who is taller?**

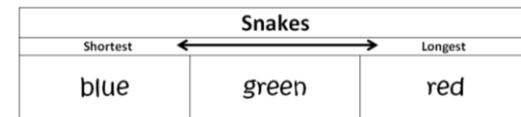
**Student:** Let’s stand back to back and compare our heights. Look! I’m taller!

Example: **Find at least 3 objects in the classroom that are the same length as, longer than, and shorter than your forearm.**

Sometimes, a third object can be used as an intermediary, allowing *indirect* comparison. For example, if we know that Aleisha is taller than Barbara and that Barbara is taller than Callie, then we know (due to the transitivity of “taller than”) that Aleisha is taller than Callie, even if Aleisha and Callie never stand back to back. This concept is referred to as the transitivity principle for indirect measurement.

Example: **The snake handler is trying to put the snakes in order- from shortest to longest. She knows that the red snake is longer than the green snake. She also knows that the green snake is longer than the blue snake. What order should she put the snakes?**

**Student:** Ok. I know that the red snake is longer than the green snake and the blue snake because, since it’s longer than the green, that means that it’s also longer than the blue snake. So the longest snake is the red snake. I also know that the green snake and red snake are both longer than the blue snake. So, the blue snake is the shortest snake. That means that the green snake is the medium sized snake.



NOTE: The Transitivity Principle (“transitivity”)<sup>1</sup>: If the length of object A is greater than the length of object B, and the length of object B is greater than the length of object C, then the length of object A is greater than the length of object C. This principle applies to measurement of other quantities as well.

Example: **Which is longer: the height of the bookshelf or the height of a desk?**

**Student A:** I used a pencil to measure the height of the bookshelf and it was 6 pencils long. I used the same pencil to measure the height of the desk and the desk was 4 pencils long. Therefore, the bookshelf is taller than the desk.

**Student B:** I used a book to measure the bookshelf and it was 3 books long. I used the same book to measure the height of the desk and it was a little less than 2 books long. Therefore, the bookshelf is taller than the desk.

Another important set of skills and understandings is ordering a set of objects by length. Such sequencing requires multiple comparisons (no more than 6 objects). Students need to understand that each object in a seriation is larger than those that come before it, and shorter than those that come after.

Example: **The snake handler is trying to put the snakes in order- from shortest to longest. Here are the three snakes (3 strings of different length and color). What order should she put the snakes?**

**Student:** Ok. I will lay the snakes next to each other. I need to make sure to be careful and line them up so they all start at the same place. So, the blue snake is the shortest. The green snake is the longest. And the red snake is medium-sized. So, I’ll put them in order from shortest to longest: blue, red, green.

*(Progressions for CCSSM: Geometric Measurement, The CCSS Writing Team, June 2012.)*

**1.MD.2** Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. *Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.*

First Graders use objects to measure items to help students focus on the attribute being measured. Objects also lends itself to future discussions regarding the need for a standard unit.

First Grade students use multiple copies of one object to measure the length larger object. They learn to lay physical units such as centimeter or inch manipulatives end-to-end and count them to measure a length. Through numerous experiences and careful questioning by the teacher, students will recognize the importance of careful measuring so that there are not any gaps or overlaps in order to get an accurate measurement. This concept is a foundational building block for the concept of area in 3<sup>rd</sup> Grade.

Example: **How long is the pencil, using paper clips to measure?**

**Student:** I carefully placed paper clips end to end.  
The pencil is 5 paper clips long. I thought it would take about 6 paperclips.



When students use different sized units to measure the same object, they learn that the sizes of the units must be considered, rather than relying solely on the amount of objects counted.

Example: **Which row is longer?**



**Student Incorrect Response:** The row with 6 sticks is longer. Row B is longer.

**Student Correct Response:** They are both the same length. See, they match up end to end.

In addition, understanding that the results of measurement and direct comparison have the same results encourages children to use measurement strategies.

Example: **Which string is longer? Justify your reasoning.**

**Student:** I placed the two strings side by side. The red string is longer than the blue string. But, to make sure, I used color tiles to measure both strings. The red string measured 8 color tiles. The blue string measure 6 color tiles. So, I was right. The red string is longer.

**NOTE:** The instructional progression for teaching measurement begins by ensuring that students can perform direct comparisons. Then, children should engage in experiences that allow them to connect number to length, using manipulative units that have a standard unit of length, such as centimeter cubes. These can be labeled “length-units” with the students. Students learn to lay such physical units end-to-end and count them to measure a length. They compare the results of measuring to direct and indirect comparisons.

*(Progressions for CCSSM: Geometric Measurement, The CCSS Writing Team, June 2012.)*

## Common Core Cluster

### Tell and write time.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **time, hour, half-hour, about, o'clock, past, "six"-thirty, analog clock, digital clock**

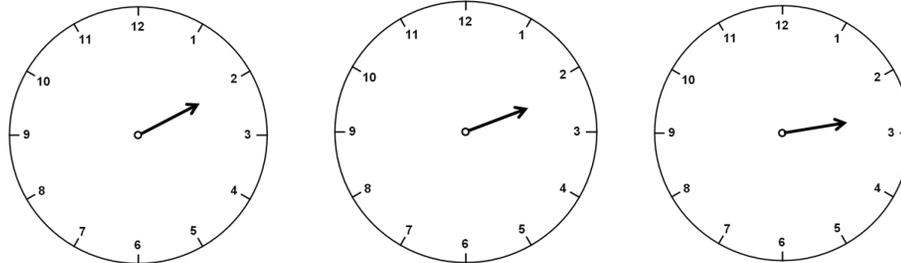
#### Common Core Standard

**1.MD.3** Tell and write time in hours and half-hours using analog and digital clocks.

#### Unpacking

What do these standards mean a child will know and be able to do?

For young children, reading a clock can be a difficult skill to learn. In particular, they must understand the differences between the two hands on the clock and the functions of these hands. By carefully watching and talking about a clock with only the hour hand, First Graders notice when the hour hand is directly pointing at a number, or when it is slightly ahead/behind a number. In addition, using language, such as "about 5 o'clock" and "a little bit past 6 o'clock", and "almost 8 o'clock" helps children begin to read an hour clock with some accuracy. Through rich experiences, First Grade students read both analog (numbers and hands) and digital clocks, orally tell the time, and write the time to the hour and half-hour.



All of these clocks indicate the hour of "two", although they look slightly different. This is an important idea for students as they learn to tell time.

## Common Core Cluster

### Represent and interpret data.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **data, more, most, less, least, same, different, category, question, collect**

#### Common Core Standard

**1.MD.4** Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.

#### Unpacking

What do these standards mean a child will know and be able to do?

First Grade students collect and use categorical data (e.g., eye color, shoe size, age) to answer a question. The data collected are often organized in a chart or table. Once the data are collected, First Graders interpret the data to determine the answer to the question posed. They also describe the data noting particular aspects such as the total number of answers, which category had the most/least responses, and interesting differences/similarities between the categories. As the teacher provides numerous opportunities for students to create questions, determine up to 3 categories of possible responses, collect data, organize data, and interpret the results, First Graders build a solid foundation for future data representations (picture and bar graphs) in Second Grade.

#### Example: Survey Station

During Literacy Block, a group of students work at the Survey Station. Each student writes a question, creates up to 3 possible answers, and walks around the room collecting data from classmates. Each student then interprets the data and writes 2-4 sentences describing the results. When all of the students in the Survey Station have completed their own data collection, they each share with one another what they discovered. They ask clarifying questions of one another regarding the data, and make revisions as needed. They later share their results with the whole class.

**Student:** The question, “What is your favorite flavor of ice cream?” is posed and recorded. The categories chocolate, vanilla and strawberry are determined as anticipated responses and written down on the recording sheet. When asking each classmate about their favorite flavor, the student’s name is written in the appropriate category. Once the data are collected, the student counts up the amounts for each category and records the amount. The student then analyzes the data by carefully looking at the data and writes 4 sentences about the data.

Name: Barbara

What is your favorite flavor of ice cream?	
Chocolate	Amy Ethan Dylan Emma Ryan Elijah Ava Emily Aiden Brittany THOMAS Nathan 12
Vanilla	Sarah Maria Brian Katie KITTY 5
Strawberry	Rodney Brandon Darrell Mia Tonya Jose 6

12 people liked chocolate. Chocolate has the most votes. Vanilla has 5 votes. 1 more vote and it can tie with strawberry.

**Common Core Cluster**

**Reason with shapes and their attributes.**

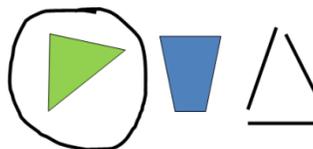
Students compose and decompose plane or solid figures (e.g., put two triangles together to make a quadrilateral) and build understanding of part-whole relationships as well as the properties of the original and composite shapes. As they combine shapes, they recognize them from different perspectives and orientations, describe their geometric attributes, and determine how they are alike and different, to develop the background for measurement and for initial understandings of properties such as congruence and symmetry.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **shape, closed, open, side, attribute<sup>1</sup>, feature<sup>1</sup>, two-dimensional, rectangle, square, trapezoid, triangle, half-circle, and quarter-circle, three-dimensional, cube, cone, prism, cylinder, partition, equal shares, halves, fourths, quarters, half of, fourth of, quarter of**

From previous grades: **circle, rectangle, hexagon, sphere, cube, cone, cylinder**

<sup>1</sup> “**Attributes**” and “**features**” are used interchangeably to indicate any characteristic of a shape, including properties, and other defining characteristics (e.g., straight sides) and non-defining characteristics (e.g., “right-side up”). (*Progressions for the CCSSM: Geometry*, CCSS Writing Team, August 2011, page 3 footnote)

Common Core Standards	Unpacking What do these standards mean a child will know and be able to do?
<p><b>1.G.1</b> Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes.</p>	<p>First Grade students use their beginning knowledge of defining and non-defining attributes of shapes to identify, name, build and draw shapes (including triangles, squares, rectangles, and trapezoids). They understand that defining attributes are always-present features that classify a particular object (e.g., number of sides, angles, etc.). They also understand that non-defining attributes are features that may be present, but do not identify what the shape is called (e.g., color, size, orientation, etc.).</p> <p><u>Example:</u> All triangles must be closed figures and have 3 sides. These are defining attributes. Triangles can be different colors, sizes and be turned in different directions. These are non-defining attributes.</p> <p><b>Student</b> I know that this shape is a triangle because it has 3 sides. It’s also closed, not open.</p> <p><b>Student</b> I used toothpicks to build a square. I know it’s a square because it has 4 sides. And, all 4 sides are the same size.</p> <p><u>TEACHER NOTE:</u> In the U.S., the term “trapezoid” may have two different meanings. Research identifies these as inclusive and exclusive definitions. The inclusive definition states: A trapezoid is a quadrilateral with <i>at least</i> one pair of parallel sides. The exclusive definition states: <b>A trapezoid is a quadrilateral with exactly one pair of parallel sides.</b> With</p>



this definition, a parallelogram is not a trapezoid. North Carolina has adopted the exclusive definition. (*Progressions for the CCSSM: Geometry*, The Common Core Standards Writing Team, June 2012.)

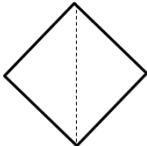
**1.G.2** Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape.<sup>1</sup>

<sup>1</sup> Students do not need to learn formal names such as “right rectangular prism.”

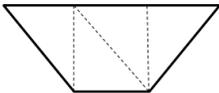
As first graders create composite shapes, a figure made up of two or more geometric shapes, they begin to see how shapes fit together to create different shapes. They also begin to notice shapes within an already existing shape. They may use such tools as pattern blocks, tangrams, attribute blocks, or virtual shapes to compose different shapes.

Example: **What shapes can you create with triangles?**

**Student A:** I made a square. I used 2 triangles.



**Student B:** I made a trapezoid. I used 4 triangles.



**Student C:** I made a tall skinny rectangle. I used 6 triangles.



First graders learn to perceive a combination of shapes as a single new shape (e.g., recognizing that two isosceles triangles can be combined to make a rhombus, and simultaneously seeing the rhombus and the two triangles). Thus, they develop competencies that include:

- Solving shape puzzles
- Constructing designs with shapes
- Creating and maintaining a shape as a unit

As students combine shapes, they continue to develop their sophistication in describing geometric attributes and properties and determining how shapes are alike and different, building foundations for measurement and initial understandings of properties such as congruence and symmetry.

(*Progressions for the CCSS in Mathematics: Geometry*, The Common Core Standards Writing Team, June 2012)

**1.G.3** Partition circles and rectangles into two and four equal shares, describe the shares using the words *halves*, *fourths*, and *quarters*, and use the phrases *half of*, *fourth of*, and

First Graders begin to partition regions into equal shares using a context (e.g., cookies, pies, pizza). This is a foundational building block of fractions, which will be extended in future grades. Through ample experiences with multiple representations, students use the words, *halves*, *fourths*, and *quarters*, and the phrases *half of*, *fourth of*, and *quarter of* to describe their thinking and solutions. Working with the “the whole”, students understand that “the whole” is composed of two halves, or four fourths or four quarters.

Students need many experiences with different sized circles and rectangles to recognize that when they cut something into two equal pieces, each piece will equal one half of its original whole. Children should recognize that halves of two different

*quarter of.* Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares.

wholes are not necessarily the same size. Also they should reason that decomposing equal shares into more equal shares results in smaller equal shares.

**Example:** How can you and a friend share equally (partition) this piece of paper so that you both have the same amount of paper to paint a picture?



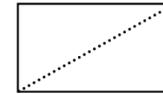
**Student 1**

I would split the paper right down the middle. That gives us 2 halves. I have half of the paper and my friend has the other half of the paper.



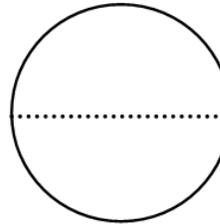
**Student 2**

I would split it from corner to corner (diagonally). She gets half of the paper and I get half of the paper. See, if we cut on the line, the parts are the same size.



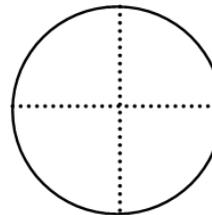
**Example:** Let's take a look at this pizza.

**Teacher:** There is pizza for dinner. What do you notice about the slices on the pizza?



**Student:** There are two slices on the pizza. Each slice is the same size. Those are big slices!

**Teacher:** If we cut the same pizza into four slices (fourths), do you think the slices would be the same size, larger, or smaller as the slices on this pizza?



**Student:** When you cut the pizza into fourths, the slices are smaller than the other pizza. More slices mean that the slices get

smaller and smaller. I want a slice from that first pizza!

## Glossary

Table 1 Common addition and subtraction situations<sup>1</sup>

<b>Add to</b>	<b>Result Unknown</b> Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? $2 + 3 = ?$ <b>(K)</b>	<b>Change Unknown</b> Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? $2 + ? = 5$ <b>(1<sup>st</sup>)</b>	<b>Start Unknown</b> Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? $? + 3 = 5$ <b>One-Step Problem (2<sup>nd</sup>)</b>
	<b>Take from</b> Five apples were on the table. I ate two apples. How many apples are on the table now? $5 - 2 = ?$ <b>(K)</b>	<b>Change Unknown</b> Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? $5 - ? = 3$ <b>(1<sup>st</sup>)</b>	<b>Start Unknown</b> Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? $? - 2 = 3$ <b>One-Step Problem (2<sup>nd</sup>)</b>
<b>Put Together/ Take Apart<sup>3</sup></b>	<b>Total Unknown</b> Three red apples and two green apples are on the table. How many apples are on the table? $3 + 2 = ?$ <b>(K)</b>	<b>Addend Unknown</b> Five apples are on the table. Three are red and the rest are green. How many apples are green? $3 + ? = 5, 5 - 3 = ?$ <b>(1<sup>st</sup>)</b>	<b>Both Addends Unknown<sup>2</sup></b> Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? $5 = 0 + 5, 5 = 5 + 0$ $5 = 1 + 4, 5 = 4 + 1$ $5 = 2 + 3, 5 = 3 + 2$ <b>(K)</b>
	<b>Compare<sup>4</sup></b>	<b>Difference Unknown</b> ("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? <b>(1<sup>st</sup>)</b>	<b>Bigger Unknown</b> (Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? <b>One-Step Problem (1<sup>st</sup>)</b>
<b>Difference Unknown</b> ("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? $2 + ? = 5, 5 - 2 = ?$ <b>(1<sup>st</sup>)</b>		<b>Bigger Unknown</b> (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? $2 + 3 = ?, 3 + 2 = ?$ <b>One-Step Problem (2<sup>nd</sup>)</b>	<b>Smaller Unknown</b> (Version with "fewer"): Lucy has three fewer apples than Julie. Julie has five apples. How many apples does Lucy have? <b>One-Step Problem (1<sup>st</sup>)</b>

K: Problem types to be mastered by the end of the Kindergarten year.

**1st:** Problem types to be mastered by the end of the First Grade year, including problem types from the previous year(s). However, First Grade students should have experiences with all 12 problem types.

**2nd:** Problem types to be mastered by the end of the Second Grade year, including problem types from the previous year(s).

1Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).

2These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean makes or results in but always does mean is the same number as.

3Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation, especially for small numbers less than or equal to 10.

4For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult.

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